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Structures of the $f_0(980)$, $a_0(980)$ mesons and the strong coupling constants $g_{f_0K^+K^-}$, $g_{a_0K^+K^-}$ with light-cone QCD sum rules

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Abstract. Making the assumption of explicit isospin violation arising from $f_0(980)$–$a_0(980)$ mixing, we take the point of view that the scalar mesons $f_0(980)$ and $a_0(980)$ have both strange and non-strange quark–antiquark components and evaluate the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ within the framework of the light-cone QCD sum rules approach. The large strong scalar–KK couplings through both the $n\bar{n}$ and $s\bar{s}$ components $g^{\bar{n}n}_{f_0K^+K^-}$, $g^{\bar{s}s}_{f_0K^+K^-}$, $g^{\bar{n}n}_{a_0K^+K^-}$ and $g^{\bar{s}s}_{a_0K^+K^-}$ will support the hadronic dressing mechanism; furthermore, in spite of the constituent structure differences between the $f_0(980)$ and $a_0(980)$ mesons, the strange components have larger strong coupling constants with the K^+K^- state than the corresponding non-strange ones, $g^{\bar{s}s}_{f_0K^+K^-} \approx \sqrt{2}g^{\bar{n}n}_{f_0K^+K^-}$ and $g^{\bar{s}s}_{a_0K^+K^-} \approx \sqrt{2}g^{\bar{n}n}_{a_0K^+K^-}$. From the existing controversial values, we cannot reach a general consensus on the strong coupling constants $g_{f_0K^+K^-}$, $g_{a_0K^+K^-}$ and the mixing angles.

1 Introduction

The constituent quark model provides a rather successful description of the spectrum of the mesons in terms of quark–antiquark bound states, which fit into the suitable multiplets reasonably well. However, the scalar mesons present a remarkable exception, as the structures of those mesons have not yet been unambiguously determined [1, 2]. From the point of view of experiment, the broad width (for the $f_0(980)$, $a_0(980)$ et cetera the widths are comparatively narrow) and strong overlaps with the continuum background make those particles difficult to resolve. On the other hand, the numerous candidates with the same quantum numbers for the quark–antiquark ($q\bar{q}$) scalar states obviously exceed the prediction power of the constituent quark model in the energy region below 2 GeV; for example, the isospin $I = 0$ scalars $f_0(400–1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ cannot be accommodated in one $q\bar{q}$ nonet, and some are supposed to be glueball, molecule, multi-quark state, et cetera. In fact, the light scalar mesons are the subject of an intense and continuous controversy in clarifying the hadron spectroscopy; the more elusive things are the constituent structures of the $f_0(980)$ and $a_0(980)$ mesons with almost degenerate masses. In the naive constituent quark model, the isovector $a_0(980)$ meson is interpreted as $a_0 = (u\bar{u}-d\bar{d})/\sqrt{2}$ and the isoscalar $f_0(980)$ meson is taken as the pure $s\bar{s}$ state, $f_0 = s\bar{s}$; while the four quark $qq\bar{q}\bar{q}$ state suggestions propose that the $f_0(980)$ and $a_0(980)$ mesons could either be compact objects i.e. nucleon-like bound states of quarks with symbolic quark structures $f_0 = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$ and $a_0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$ [3], or spatially extended objects i.e. deuteron-like bound states of hadrons; for example, the $f_0(980)$ meson is usually taken as a $K\bar{K}$ molecule, et cetera [4]. The hadronic dressing mechanism takes the point of view that the $f_0(980)$ and $a_0(980)$ mesons have small $q\bar{q}$ cores of typical $q\bar{q}$ meson size, and the strong couplings to the hadronic channels enrich the pure $q\bar{q}$ states with other components and spend part (or most part) of their lifetime as virtual $K\bar{K}$ states [5]. Despite what constituents they may have, we have the fact that they both lie just a little below the $K\bar{K}$ threshold, and the strong interactions with the $K\bar{K}$ threshold will significantly influence their dynamics. In strong interactions (QCD), the isospin is believed to be a nearly exact symmetry, broken only by the slight mass difference between the u and d quarks, or electroweak effects; however, the mass gaps between the $f_0(980)$, $a_0(980)$ and the K^+K^- and $K^0\bar{K}^0$ thresholds make an exception and cannot be explained. The mixing of the two scalar mesons i.e. the broken isospin can occur through the transitions between the intermediate K^+K^- and $K^0\bar{K}^0$ states. In [6], an analysis of the central production in the reaction $pp \to p_s(\eta\pi^0)p_f$ shows

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that the $f_0(980)$ and $a_0(980)$ mesons can mix substantially with each other with an intensity of about $(8\pm3)\%$ and the isospin symmetry is obviously broken. The isospin mixing effects could considerably alter some existing predictions for the radiative decays $\phi\to f_0\gamma$ and $\phi\to a_0\gamma$ [7]; however, further studies show that when the physical masses and widths are included, the mixing effects are very small [8]. The vector meson dominance model also indicates that the mixing effects are small [9]. On the other hand, the generalized Jülich meson exchange model for $\pi\pi, K\bar{K}, \pi\eta$ scattering with physical mass eigenstates predicts that the charged and neutral K mass splitting induced isospin violation and the coupled $\pi\pi$–$K\bar{K}$ channels induced G-parity violation give rise to a non-vanishing cross section for the $\pi\pi$–$\pi^0\eta$ transition and lead to $f_0(980)$–$a_0(980)$ mixing [10]. In [11], the authors suggest to perform the polarized target experiments on the reaction $\pi^-p\to\eta\pi^0n$ at high energy in which the existence of $a_0^0(980)$–$f_0(980)$ mixing can be unambiguously and very easily established through the presence of a strong jump in the azimuthal asymmetry of the $\eta\pi^0$ S wave production cross section near the $K\bar{K}$ thresholds. If we take the $f_0(980)$–$a_0(980)$ mixing and explicit isospin violation for granted, no matter how tiny they are, we can take the point of view that the $f_0(980)$ and $a_0(980)$ mesons both have two possible constituent $q\bar{q}$ states, i.e., $n\bar{n}$ and $s\bar{s}$ in the $q\bar{q}$ quark model; in the isospin limit, the $a_0(980)$ meson has a pure $n\bar{n}$ quark structure, $a_0=\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$, with isospin $I=1$, and cannot have an $s\bar{s}$ component, while the isospin $I=0$ meson $f_0(980)$ can have both $n\bar{n}=\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ and $s\bar{s}$ components. The K-matrix analysis of the channels $f_0\to\pi\pi,\pi\pi\pi\pi,K\bar{K},\eta\eta,\eta\eta'$ shows that the $f_0(980)$ meson may have both $n\bar{n}$ and $s\bar{s}$ components, even the gluonium component [12]. The radiative decays of the $\phi(1020)$ meson $\phi(\to K^+K^-)\to a_0\gamma\to\gamma\pi\eta$ and $\phi(\to K^+K^-)\to f_0\gamma\to\gamma\pi\pi$ provide an efficient tool to investigate the structures of the $a_0(980)$ and $f_0(980)$ mesons. It is generally agreed that the experimental data support the $K\bar{K}$ mesons loop mechanism for those decays, where the radiative decays occur through the photon emission from the intermediate K^+K^- loop. The important hadronic parameters entering the analysis involving the $f_0(980)$ and $a_0(980)$ mesons are the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$.

In this article, we take the point of view that the $f_0(980)$ and $a_0(980)$ mesons are mixed states which consist of both $n\bar{n}$ and $s\bar{s}$ components, and it is devoted to a determination of the values of the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ within the framework of the light-cone QCD sum rules approach, by carrying out the operator product expansion near the light-cone, $x^2\approx0$, instead of the short distance, $x\approx0$, while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes which are classified according to their twists instead of the vacuum condensates [13–15].

This article is arranged as follows: in Sect. 2, the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ are evaluated with the light-cone QCD sum rules approach, and in Sect. 3, one finds a conclusion.

2 Strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ with light-cone QCD sum rules

In the following, we write down the definitions for the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$:

$$\langle K^+(q)K^-(p)|f_0(p+q)\rangle = g_{f_0K^+K^-}\,,$$
$$\langle K^+(q)K^-(p)|a_0(p+q)\rangle = g_{a_0K^+K^-}\,. \quad (1)$$

In this article, we investigate the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ with the scalar interpolating currents J_{f_0} and J_{a_0} and choose the following two two-point correlation functions:

$$J_{f_0}=\sin\theta\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}+\cos\theta\bar{s}s, \quad (2)$$

$$J_{a_0}=\sin\varphi\frac{\bar{u}u-\bar{d}d}{\sqrt{2}}+\cos\varphi\bar{s}s, \quad (3)$$

$$T_\mu^{f_0}(p,q)=\mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}p\cdot x}\,\langle K^+(q)|T[J_\mu(x)J_{f_0}(0)]|0\rangle, \quad (4)$$

$$T_\mu^{a_0}(p,q)=\mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}p\cdot x}\,\langle K^+(q)|T[J_\mu(x)J_{a_0}(0)]|0\rangle. \quad (5)$$

Here the axial-vector current $J_\mu=\bar{u}\gamma_\mu\gamma_5 s$ interpolates the pseudoscalar K meson, and the external K state has four-momentum q with $q^2=M_K^2$. If the isospin violation is small, the parameter φ is close to $\frac{\pi}{2}$. Those correlation functions in (4) and (5) can be decomposed as follows:

$$T_\mu^{f_0}(p,q)=T_p^{f_0}\left(p^2,(p+q)^2\right)p_\mu + T_q^{f_0}\left(p^2,(p+q)^2\right)q_\mu, \quad (6)$$
$$T_\mu^{a_0}(p,q)=T_p^{a_0}\left(p^2,(p+q)^2\right)p_\mu + T_q^{a_0}\left(p^2,(p+q)^2\right)q_\mu, \quad (7)$$

due to tensor analysis.

With the basic assumption of hadron–quark duality of the QCD sum rules approach [16], we can insert a complete series of intermediate states with the same quantum numbers as the current operators J_{f_0} , J_{a_0} and J_μ into those correlation functions in (4) and (5) to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the $f_0(980)$, $a_0(980)$ and K mesons, we get the following result:

$$T_p^{f_0}\left(p^2,(p+q)^2\right)p_\mu = \frac{\langle 0\mid J_\mu\mid K(p)\rangle\langle KK\mid f_0\rangle\langle f_0(p+q)|J_{f_0}\mid 0\rangle}{(M_K^2-p^2)\left[M_{f_0}^2-(p+q)^2\right]}+\cdots = \frac{\mathrm{i}f_K g_{f_0K^+K^-}f_{f_0}M_{f_0}p_\mu}{(M_K^2-p^2)\left[M_{f_0}^2-(p+q)^2\right]}+\cdots\,; \quad (8)$$

$$T_p^{a_0}\left(p^2,(p+q)^2\right)p_\mu$$

$$= \frac{\langle 0 \mid J_\mu \mid K(p)\rangle \langle KK \mid a_0\rangle \langle a_0(p+q)|J_{a_0} \mid 0\rangle}{(M_K^2-p^2)\left[M_{a_0}^2-(p+q)^2\right]} + \cdots$$
$$= \frac{\mathrm{i} f_K g_{a_0K^+K^-} f_{a_0} M_{a_0} p_\mu}{(M_K^2-p^2)\left[M_{a_0}^2-(p+q)^2\right]} + \cdots , \quad (9)$$

where the following definitions have been used:

$$\langle f_0(p+q) \mid J_{f_0} \mid 0\rangle = f_{f_0} M_{f_0} ,$$
$$\langle a_0(p+q) \mid J_{a_0} \mid 0\rangle = f_{a_0} M_{a_0} ,$$
$$\langle 0 \mid J_\mu \mid K(p)\rangle = \mathrm{i} f_K p_\mu . \quad (10)$$

Here we have not shown the contributions from the higher resonances and continuum states explicitly as they are suppressed due to the Borel transformation. In the ground state approximation, the tensor structures $T_q^{f_0}\left(p^2,(p+q)^2\right) q_\mu$ and $T_q^{a_0}\left(p^2,(p+q)^2\right) q_\mu$ have no contributions and are neglected.

In the following, we briefly outline the operator product expansion for the correlation functions in (4) and (5) in perturbative QCD theory. The calculations are performed at large space-like momentum regions $(p+q)^2 \ll 0$ and $p^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. Firstly, let us write down the propagator of a massive quark in the external gluon field in the Fock–Schwinger gauge [17,18]:

$$\langle 0|T\{q_i(x_1)\,\bar{q}_j(x_2)\}|0\rangle$$
$$= \mathrm{i} \int \frac{\mathrm{d}^4k}{(2\pi)^4} \mathrm{e}^{-\mathrm{i}k(x_1-x_2)}$$
$$\times \left\{ \frac{\not k + m}{k^2-m^2}\delta_{ij} - \int_0^1 \mathrm{d}v\, g_\mathrm{s}\, G_a^{\mu\nu}(vx_1+(1-v)x_2)\left(\frac{\lambda^a}{2}\right)_{ij} \right.$$
$$\left. \times \left[\frac{1}{2}\frac{\not k + m}{(k^2-m^2)^2}\sigma_{\mu\nu} - \frac{1}{k^2-m^2}v(x_1-x_2)_\mu\gamma_\nu\right]\right\}; \quad (11)$$

here $G_a^{\mu\nu}$ is the gluonic field strength, and g_s denotes the strong coupling constant. Substituting the above u, s quark propagators and the corresponding K meson light-cone distribution amplitudes into (4) and (5) and completing the integrals over x and k, we finally obtain

$$T_p^{f_0}(p^2,(p+q)^2)$$
$$= \sin\theta \frac{1}{\sqrt{2}}\left\{ \mathrm{i} f_K \int_0^1 \mathrm{d}u \left\{ \frac{M_K^2}{m_s}\varphi_p(u)\frac{1}{-(p+uq)^2} \right.\right.$$
$$\left. - 2\frac{M_K^2}{6m_s}\varphi_\sigma(u)(p\cdot q+uM_K^2)\frac{1}{[-(p+uq)^2]^2}\right\}$$
$$+ \mathrm{i} f_{3K} M_K^2 \int_0^1 \mathrm{d}v\,(2v-3)\int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i)$$
$$\left.\times \frac{1}{\{[p+q(\alpha_1+v\alpha_3)]^2\}^2}\right\}$$
$$+ \cos\theta \left\{ \mathrm{i} f_K \int_0^1 \mathrm{d}u \left\{ \frac{M_K^2}{m_s}\varphi_p(u)\frac{1}{m_s^2-(p+uq)^2} \right.\right.$$
$$- 2\left[m_s g_2(u) + \frac{M_K^2}{6m_s}\varphi_\sigma(u)(p\cdot q+uM_K^2)\right]$$
$$\left.\times \frac{1}{[m_s^2-(p+uq)^2]^2}\right\}$$
$$+ \mathrm{i} f_{3K} M_K^2 \int_0^1 \mathrm{d}v\,(2v-3)\int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i)$$
$$\times \frac{1}{\{[p+q(\alpha_1+v\alpha_3)]^2-m_s^2\}^2}$$
$$- 4\mathrm{i} f_K m_s M_K^2 \left\{ \int_0^1 \mathrm{d}v(v-1)\int_0^1 \mathrm{d}\alpha_3 \int_0^{\alpha_3}\mathrm{d}\beta \int_0^{1-\beta}\mathrm{d}\alpha \right.$$
$$\times \frac{\Phi(\alpha,1-\alpha-\beta,\beta)}{\{[p+(1-\alpha_3+v\alpha_3)q]^2-m_s^2\}^3}$$
$$+ \int_0^1 \mathrm{d}v \int_0^1 \mathrm{d}\alpha_3 \int_0^{1-\alpha_3}\mathrm{d}\alpha_1 \int_0^{\alpha_1}\mathrm{d}\alpha$$
$$\left.\left. \times \frac{\Phi(\alpha,1-\alpha-\alpha_3,\alpha_3)}{\{[p+(\alpha_1+v\alpha_3)q]^2-m_s^2\}^3}\right\}\right\}; \quad (12)$$

$$T_p^{a_0}(p^2,(p+q)^2)$$
$$= \sin\varphi \frac{1}{\sqrt{2}}\left\{ \mathrm{i} f_K \int_0^1 \mathrm{d}u \left\{ \frac{M_K^2}{m_s}\varphi_p(u)\frac{1}{-(p+uq)^2} \right.\right.$$
$$\left. - 2\frac{M_K^2}{6m_s}\varphi_\sigma(u)(p\cdot q+uM_K^2)\frac{1}{[-(p+uq)^2]^2}\right\}$$
$$+ \mathrm{i} f_{3K} M_K^2 \int_0^1 \mathrm{d}v\,(2v-3)\int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i)$$
$$\left.\times \frac{1}{\{[p+q(\alpha_1+v\alpha_3)]^2\}^2}\right\}$$
$$+ \cos\varphi \left\{ \mathrm{i} f_K \int_0^1 \mathrm{d}u \left\{ \frac{M_K^2}{m_s}\varphi_p(u)\frac{1}{m_s^2-(p+uq)^2} \right.\right.$$
$$- 2\left[m_s g_2(u) + \frac{M_K^2}{6m_s}\varphi_\sigma(u)(p\cdot q+uM_K^2)\right]$$
$$\left.\times \frac{1}{[m_s^2-(p+uq)^2]^2}\right\}$$
$$+ \mathrm{i} f_{3K} M_K^2 \int_0^1 \mathrm{d}v\,(2v-3)\int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i)$$
$$\times \frac{1}{\{[p+q(\alpha_1+v\alpha_3)]^2-m_s^2\}^2}$$
$$- 4\mathrm{i} f_K m_s M_K^2 \left\{ \int_0^1 \mathrm{d}v(v-1)\int_0^1 \mathrm{d}\alpha_3 \int_0^{\alpha_3}\mathrm{d}\beta \int_0^{1-\beta}\mathrm{d}\alpha \right.$$
$$\times \frac{\Phi(\alpha,1-\alpha-\beta,\beta)}{\{[p+(1-\alpha_3+v\alpha_3)q]^2-m_s^2\}^3}$$
$$+ \int_0^1 \mathrm{d}v \int_0^1 \mathrm{d}\alpha_3 \int_0^{1-\alpha_3}\mathrm{d}\alpha_1 \int_0^{\alpha_1}\mathrm{d}\alpha$$
$$\left.\left. \times \frac{\Phi(\alpha,1-\alpha-\alpha_3,\alpha_3)}{\{[p+(\alpha_1+v\alpha_3)q]^2-m_s^2\}^3}\right\}\right\}. \quad (13)$$

In the limit $\theta = 0$, our results for the expressions of the three-particle twist-3 and twist-4 terms in (12) are slightly different from the corresponding ones in [19]; there may be some errors (or just writing errors) in their calculations. If we take the limit $\varphi = \frac{\pi}{2}$ in (13), the results for the isospin-vector scalar current are found; the expressions for the contributions from the three-particle twist-3 light-cone distribution amplitudes may have some errors (or just writing errors) in [20]. However, the contributions from those terms are small and cannot significantly affect the numerical values. Comparing with the mass of the s quark, the masses of the u and d quarks are neglected.

In the calculation, the following two-particle and three-particle K meson light-cone distribution amplitudes are useful:

$$\begin{aligned}
&\langle K(q)|\bar{u}(x)\gamma_\mu\gamma_5 s(0)|0\rangle \\
&= -\mathrm{i}f_K q_\mu \int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}uq\cdot x}[\varphi_K(u)+x^2 g_1(u)] \\
&+ f_K\left(x_\mu - \frac{q_\mu x^2}{q\cdot x}\right)\int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}uq\cdot x} g_2(u), \\
&\langle K(q)|\bar{u}(x)\mathrm{i}\gamma_5 s(0)|0\rangle = \frac{f_K M_K^2}{m_s}\int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}uq\cdot x}\varphi_p(u), \\
&\langle K(q)|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 s(0)|0\rangle \\
&= \mathrm{i}(q_\mu x_\nu - q_\nu x_\mu)\frac{f_K M_K^2}{6m_s}\int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}uq\cdot x}\varphi_\sigma(u), \\
&\langle K(q)|\bar{u}(x)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(vx)s(0)|0\rangle \\
&= \mathrm{i}f_{3K}[(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \\
&\quad \times \int \mathcal{D}\alpha_i\, \varphi_{3K}(\alpha_i)\mathrm{e}^{\mathrm{i}q\cdot x(\alpha_1+v\alpha_3)}, \\
&\langle K(q)|\bar{u}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)s(0)|0\rangle \\
&= f_K\left[q_\beta\left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q\cdot x}\right) - q_\alpha\left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q\cdot x}\right)\right] \\
&\quad \times \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i)\mathrm{e}^{\mathrm{i}q\cdot x(\alpha_1+v\alpha_3)} \\
&+ f_K\frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha)\int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i)\mathrm{e}^{\mathrm{i}q\cdot x(\alpha_1+v\alpha_3)}, \\
&\langle K(q)|\bar{u}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)s(0)|0\rangle \\
&= \mathrm{i}f_K\left[q_\beta\left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q\cdot x}\right) - q_\alpha\left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q\cdot x}\right)\right] \\
&\quad \times \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i)\mathrm{e}^{\mathrm{i}q\cdot x(\alpha_1+v\alpha_3)} \qquad (14) \\
&+ \mathrm{i}f_K\frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha)\int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i)\mathrm{e}^{\mathrm{i}q\cdot x(\alpha_1+v\alpha_3)}.
\end{aligned}$$

Here the operator $\tilde{G}_{\alpha\beta}$ is the dual of $G_{\alpha\beta}$, $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\delta\rho}G^{\delta\rho}$, $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i = \mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$, and $\Phi(\alpha_1,\alpha_2,\alpha_3) = \varphi_\perp + \varphi_\parallel - \tilde{\varphi}_\perp - \tilde{\varphi}_\parallel$.

The twist-3 and twist-4 light-cone distribution amplitudes can be parameterized as follows:

$$\begin{aligned}
\varphi_p(u,\mu) &= 1 + \left(30\eta_3 - \frac{5}{2}\rho^2\right)C_2^{\frac{1}{2}}(2u-1) \\
&+ \left(-3\eta_3\omega_3 - \frac{27}{20}\rho^2 - \frac{81}{10}\rho^2\tilde{a}_2\right)C_4^{\frac{1}{2}}(2u-1) \\
\varphi_\sigma(u,\mu) &= 6u(1-u) \\
&\times \left(1 + \left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho^2 - \frac{3}{5}\rho^2\tilde{a}_2\right)C_2^{\frac{3}{2}}\right. \\
&\left.\times (2u-1)\right), \\
\phi_{3K}(\alpha_i,\mu) \\
&= 360\alpha_1\alpha_2\alpha_3^2\left(1 + a(\mu)\frac{1}{2}(7\alpha_3-3)\right. \\
&\quad + b(\mu)(2-4\alpha_1\alpha_2-8\alpha_3(1-\alpha_3)) \\
&\quad \left. + c(\mu)(3\alpha_1\alpha_2 - 2\alpha_3 + 3\alpha_3^2)\right), \\
\phi_\perp(\alpha_i,\mu) \\
&= 30\delta^2(\mu)(\alpha_1-\alpha_2)\alpha_3^2\left[\frac{1}{3} + 2\epsilon(\mu)(1-2\alpha_3)\right], \\
\phi_{||}(\alpha_i,\mu) &= 120\delta^2(\mu)\epsilon(\mu)(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3, \\
\tilde{\phi}_\perp(\alpha_i,\mu) \\
&= 30\delta^2(\mu)\alpha_3^2(1-\alpha_3)\left[\frac{1}{3} + 2\epsilon(\mu)(1-2\alpha_3)\right], \\
\tilde{\phi}_{||}(\alpha_i,\mu) &= -120\delta^2(\mu)\alpha_1\alpha_2\alpha_3\left[\frac{1}{3} + \epsilon(\mu)(1-3\alpha_3)\right],
\end{aligned} \qquad (15)$$

where $C_2^{\frac{1}{2}}$, $C_4^{\frac{1}{2}}$ and $C_2^{\frac{3}{2}}$ are Gegenbauer polynomials. The parameters in the light-cone distribution amplitudes can be estimated from the QCD sum rules approach [21–23]. In practical manipulation, we choose $a = -2.88$, $b = 0.0$, $c = 0.0$, $\delta^2 = 0.2\,\mathrm{GeV}^2$ and $\epsilon = 0.5$ at $\mu = 1\,\mathrm{GeV}$. Furthermore, the updated values for η_3, ω_3, ρ and $\tilde{a}_2$ are taken as $\tilde{a}_2 = 0.2$, $\eta_3 = 0.015$, $\omega_3 = -3$ at the scale $\mu \simeq 1\,\mathrm{GeV}$ and the parameter $\rho^2 = \frac{m_s^2}{M_K^2}$ [17,24].

Now we perform the Borel transformation with respect to the variables $Q_1^2 = -p^2$ and $Q_2^2 = -(p+q)^2$ for the correlation functions in (8) and (9) and obtain the analytical expressions for the invariant functions in the hadronic representation:

$$\begin{aligned}
&B_{M_2^2}B_{M_1^2}T_p^{f_0}(M_1^2,M_2^2) \\
&= \mathrm{i}f_K g_{f_0K^+K^-}f_{f_0}M_{f_0}\frac{1}{M_1^2M_2^2}\mathrm{e}^{-M_K^2/M_1^2}\mathrm{e}^{-M_{f_0}^2/M_2^2} \\
&+ \frac{\mathrm{i}}{M_1^2M_2^2}\int_{s_0}^\infty \mathrm{d}s\int_{s_0'}^\infty \mathrm{d}s'\ \rho^{cont}(s,s')\mathrm{e}^{-s/M_1^2}\mathrm{e}^{-s'/M_2^2}\quad ,
\end{aligned} \qquad (16)$$

$$\begin{aligned}
&B_{M_2^2}B_{M_1^2}T_p^{a_0}(M_1^2,M_2^2) \\
&= \mathrm{i}f_K g_{a_0K^+K^-}f_{a_0}M_{a_0}\frac{1}{M_1^2M_2^2}\mathrm{e}^{-M_K^2/M_1^2}\mathrm{e}^{-M_{a_0}^2/M_2^2} \\
&+ \frac{\mathrm{i}}{M_1^2M_2^2}\int_{s_0}^\infty \mathrm{d}s\int_{s_0'}^\infty \mathrm{d}s'\ \rho^{cont}(s,s')\mathrm{e}^{-s/M_1^2}\mathrm{e}^{-s'/M_2^2};
\end{aligned} \qquad (17)$$

here we have not shown the cross terms explicitly for simplicity. In order to match the duality regions below the thresholds s_0 and s_0', we can express the correlation functions at the level of the quark–gluon degrees of freedom into the following form:

$$T_p^{f_0}(p^2,(p+q)^2) = \mathrm{i}\int \mathrm{d}s\mathrm{d}s'\frac{\rho_{\mathrm{quark}}^{f_0}(s,s')}{(s-p^2)[s'-(p+q)^2]}, \qquad (18)$$

$$T_p^{a_0}(p^2,(p+q)^2) = \mathrm{i}\int \mathrm{d}s\mathrm{d}s' \frac{\rho^{a_0}_{\mathrm{quark}}(s,s')}{(s-p^2)[s'-(p+q)^2]}. \quad (19)$$

Then it is a straightforward procedure to perform the Borel transformation with respect to the variables $Q_1 = -p^2$ and $Q_2^2 = -(p+q)^2$; however, the analytical expressions for the spectral densities $\rho^{f_0}_{\mathrm{quark}}(s,s')$, $\rho^{a_0}_{\mathrm{quark}}(s,s')$ are hard to obtain, and we have to take some approximations, as the contributions from the higher twist terms are suppressed by more powers of $\frac{1}{-p^2}$ or $\frac{1}{-(p+q)^2}$ and the continuum subtractions will not affect the results remarkably. Here we will use the expressions in (12) and (13) for the three-particle (quark–antiquark–gluon) twist-3 and twist-4 terms. As for the terms involving φ_p and φ_σ, we perform the same type trick as [17,25] and expand the amplitudes $\varphi_p(u)$ and $\varphi_\sigma(u)$ in terms of polynomials of $1-u$,

$$\varphi_p(u) + \frac{\mathrm{d}\varphi_\sigma(u)}{6\mathrm{d}u} = \sum_{k=0}^N b_k(1-u)^k; \quad (20)$$

then the variable u is changed into s, s' and the spectral densities are obtained.

After straightforward but cumbersome calculations, we obtain the final expressions for the Borel transformed correlation functions at the level of quark–gluon degrees of freedom:

$$\begin{aligned}
&B_{M_2^2}B_{M_1^2}T_p^{f_0}\\
&= \frac{\sin\theta}{\sqrt{2}}\frac{\mathrm{i}}{M_1^2M_2^2}\mathrm{e}^{-\frac{u_0(1-u_0)M_K^2}{M^2}}\\
&\times\left\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0}{M^2}\right)^i}{i!}\right]\right.\\
&+ f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\\
&\qquad\left.\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\right\}\\
&+\cos\theta\frac{\mathrm{i}}{M_1^2M_2^2}\mathrm{e}^{-\frac{m_s^2+u_0(1-u_0)M_K^2}{M^2}}\\
&\times\left\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0-m_s^2}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0-m_s^2}{M^2}\right)^i}{i!}\right]\right.\\
&-2f_Km_s\,g_2(u_0)\\
&+ f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\\
&\qquad\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\\
&-\frac{2f_Km_sM_K^2}{M^2}(1-u_0)\\
&\qquad\times\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3^2}\int_0^{\alpha_3}\mathrm{d}\beta\int_0^{1-\beta}\mathrm{d}\alpha\Phi(\alpha,1-\alpha-\beta,\beta)\\
&+\frac{2f_Km_sM_K^2}{M^2}\left[\int_0^{1-u_0}\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right.\\
&\qquad\left.+\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right]\\
&\qquad\left.\times\Phi(\alpha,1-\alpha-\alpha_1,\alpha_1)\right\}, \qquad (21)
\end{aligned}$$

$$\begin{aligned}
&B_{M_2^2}B_{M_1^2}T_p^{a_0}\\
&= \frac{\sin\varphi}{\sqrt{2}}\frac{\mathrm{i}}{M_1^2M_2^2}\mathrm{e}^{-\frac{u_0(1-u_0)M_K^2}{M^2}}\\
&\times\left\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0}{M^2}\right)^i}{i!}\right]\right.\\
&+ f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\\
&\qquad\left.\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\right\}\\
&+\cos\varphi\frac{\mathrm{i}}{M_1^2M_2^2}\mathrm{e}^{-\frac{m_s^2+u_0(1-u_0)M_K^2}{M^2}}\\
&\times\left\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0-m_s^2}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0-m_s^2}{M^2}\right)^i}{i!}\right]\right.\\
&-2f_Km_s\,g_2(u_0)\\
&+ f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\\
&\qquad\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\\
&-\frac{2f_Km_sM_K^2}{M^2}(1-u_0)\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3^2}\int_0^{\alpha_3}\mathrm{d}\beta\int_0^{1-\beta}\mathrm{d}\alpha\\
&\qquad\times\Phi(\alpha,1-\alpha-\beta,\beta)\\
&+\frac{2f_Km_sM_K^2}{M^2}\left[\int_0^{1-u_0}\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right.\\
&\qquad\left.+\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right]\\
&\qquad\left.\times\Phi(\alpha,1-\alpha-\alpha_1,\alpha_1)\right\}. \qquad (22)
\end{aligned}$$

In deriving the above expressions for $\varphi_p(u)+\frac{\mathrm{d}\varphi_\sigma(u)}{6\mathrm{d}u}$, we have neglected the terms $\sim M_K^4$. Here $u_0 = \frac{M_1^2}{M_1^2+M_2^2}$ and $M^2 = \frac{M_1^2M_2^2}{M_1^2+M_2^2}$.

The matching between (16), (17) and (21), (22) below the thresholds s_0, s_0' is straightforward and we can obtain the analytical expressions for the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$:

$$\begin{aligned}
&g_{f_0K^+K^-}\\
&= \frac{\sin\theta}{\sqrt{2}}\frac{1}{f_Kf_{f_0}M_{f_0}}\mathrm{e}^{\frac{M_{f_0}^2}{M_2^2}+\frac{M_K^2}{M_1^2}-\frac{u_0(1-u_0)M_K^2}{M^2}}\\
&\times\left\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0}{M^2}\right)^i}{i!}\right]\right.\\
&+ f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)
\end{aligned}$$

$$
\begin{aligned}
&\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\Bigg\}\\
&+\cos\theta\frac{1}{f_Kf_{f_0}M_{f_0}}\mathrm{e}^{\frac{M_{f_0}^2}{M_2^2}+\frac{M_K^2}{M_1^2}-\frac{m_s^2+u_0(1-u_0)M_K^2}{M^2}}\\
&\times\Bigg\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0-m_s^2}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0-m_s^2}{M^2}\right)^i}{i!}\right]\\
&-2f_Km_s\,g_2(u_0)\\
&+f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\\
&\quad\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\\
&-\frac{2f_Km_sM_K^2}{M^2}(1-u_0)\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3^2}\\
&\times\int_0^{\alpha_3}\mathrm{d}\beta\int_0^{1-\beta}\mathrm{d}\alpha\Phi(\alpha,1-\alpha-\beta,\beta)\\
&+\frac{2f_Km_sM_K^2}{M^2}\left[\int_0^{1-u_0}\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right.\\
&\quad\left.+\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right]\\
&\quad\times\Phi(\alpha,1-\alpha-\alpha_1,\alpha_1)\Bigg\}\\
&=\sin\theta g^{\bar nn}_{f_0K^+K^-}+\cos\theta g^{\bar ss}_{f_0K^+K^-};
\end{aligned}\tag{23}
$$

$$
\begin{aligned}
&g_{a_0K^+K^-}\\
&=\frac{\sin\varphi}{\sqrt2}\frac{1}{f_Kf_{a_0}M_{a_0}}\mathrm{e}^{\frac{M_{a_0}^2}{M_2^2}+\frac{M_K^2}{M_1^2}-\frac{u_0(1-u_0)M_K^2}{M^2}}\\
&\times\left\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0}{M^2}\right)^i}{i!}\right]\right.\\
&+f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\\
&\quad\left.\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\right\}\\
&+\cos\varphi\frac{1}{f_Kf_{a_0}M_{a_0}}\mathrm{e}^{\frac{M_{a_0}^2}{M_2^2}+\frac{M_K^2}{M_1^2}-\frac{m_s^2+u_0(1-u_0)M_K^2}{M^2}}\\
&\times\Bigg\{\frac{f_KM^2M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left[1-\mathrm{e}^{-\frac{s_0-m_s^2}{M^2}}\sum_{i=0}^k\frac{\left(\frac{s_0-m_s^2}{M^2}\right)^i}{i!}\right]\\
&-2f_Km_s\,g_2(u_0)\\
&+f_{3K}M_K^2\int_0^{u_0}\mathrm{d}\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{\mathrm{d}\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\\
&\quad\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)\\
&-\frac{2f_Km_sM_K^2}{M^2}(1-u_0)\\
&\times\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3^2}\int_0^{\alpha_3}\mathrm{d}\beta\int_0^{1-\beta}\mathrm{d}\alpha\Phi(\alpha,1-\alpha-\beta,\beta)\\
&+\frac{2f_Km_sM_K^2}{M^2}\left[\int_0^{1-u_0}\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right.\\
&\quad\left.+\int_{1-u_0}^1\frac{\mathrm{d}\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3}\mathrm{d}\alpha_1\int_0^{\alpha_1}\mathrm{d}\alpha\right]\\
&\quad\times\Phi(\alpha,1-\alpha-\alpha_1,\alpha_1)\Bigg\}\\
&=\sin\varphi g^{\bar nn}_{a_0K^+K^-}+\cos\varphi g^{\bar ss}_{a_0K^+K^-}.
\end{aligned}\tag{24}
$$

The values of the parameters f_{f_0} and f_{a_0} can be determined from the conventional QCD sum rules approach with the following two two-point correlation functions:

$$
\begin{aligned}
T_{f_0}&=\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}px}\langle|T[J_{f_0}(x)J_{f_0}(0)]|\rangle\\
&=\sin^2\theta\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}px}\langle|T\left[\frac{\bar uu+\bar dd}{\sqrt2}(x)\frac{\bar uu+\bar dd}{\sqrt2}(0)\right]|\rangle\\
&+\cos^2\theta\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}px}\langle|T[\bar ss(x)\bar ss(0)]|\rangle,
\end{aligned}\tag{25}
$$

$$
\begin{aligned}
T_{a_0}&=\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}px}\langle|T[J_{a_0}(x)J_{a_0}(0)]|\rangle\\
&=\sin^2\varphi\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}px}\langle|T\left[\frac{\bar uu-\bar dd}{\sqrt2}(x)\frac{\bar uu-\bar dd}{\sqrt2}(0)\right]|\rangle\\
&+\cos^2\varphi\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}px}\langle|T[\bar ss(x)\bar ss(0)]|\rangle\,.
\end{aligned}\tag{26}
$$

The operator product expansion near $x\sim0$ in perturbative QCD is straightforward and we will not write down the detailed routine for simplicity. The final expressions for T_{f_0} and T_{a_0} at the level of quark–gluon degrees of freedom can be written as

$$
\begin{aligned}
T_{f_0}&=\sin^2\theta A(p^2)+\cos^2\theta B(p^2),\\
T_{a_0}&=\sin^2\varphi C(p^2)+\cos^2\varphi D(p^2);
\end{aligned}\tag{27}
$$

here A,B,C,D are formally written. To obtain the hadronic representation, we can insert a complete series of intermediate states with the same quantum numbers as the interpolating currents J_{f_0} and J_{a_0} into the correlation functions in (25) and (26), and then isolate the ground state contributions from the $f_0(980)$ and $a_0(980)$ mesons:

$$
\begin{aligned}
T_{f_0}&=\langle|J_{f_0}(0)|f_0(p)\rangle\frac{1}{M_{f_0}^2-p^2}\langle f_0(p)|J_{f_0}(0)|\rangle+\cdots\\
&=\frac{f_{f_0}^2M_{f_0}^2}{M_{f_0}^2-p^2}+\cdots\\
&=\sin^2\theta\langle|\frac{\bar uu+\bar dd}{\sqrt2}(0)|f_0(p)\rangle\frac{1}{M_{f_0}^2-p^2}\\
&\qquad\times\langle f_0(p)|\frac{\bar uu+\bar dd}{\sqrt2}(0)|\rangle\\
&+\cos^2\theta\langle|\bar ss(0)|f_0(p)\rangle\frac{1}{M_{f_0}^2-p^2}\langle f_0(p)|\bar ss(0)|\rangle+\cdots\\
&=\sin^2\theta\frac{f_{\bar nnf_0}^2M_{f_0}^2}{M_{f_0}^2-p^2}+\cos^2\theta\frac{f_{\bar ssf_0}^2M_{f_0}^2}{M_{f_0}^2-p^2}+\cdots,
\end{aligned}
$$

$$
T_{a_0}=\langle|J_{a_0}(0)|a_0(p)\rangle\frac{1}{M_{a_0}^2-p^2}\langle a_0(p)|J_{a_0}(0)|\rangle+\cdots,
$$

$$= \frac{f_{a_0}^2 M_{a_0}^2}{M_{a_0}^2 - p^2} + \cdots$$
$$= \sin^2\varphi\langle|\frac{\bar{u}u - \bar{d}d}{\sqrt{2}}(0)|a_0(p)\rangle\frac{1}{M_{a_0}^2 - p^2}$$
$$\times\langle a_0(p)|\frac{\bar{u}u - \bar{d}d}{\sqrt{2}}(0)|\rangle$$
$$+ \cos^2\varphi\langle|\bar{s}s(0)|a_0(p)\rangle\frac{1}{M_{a_0}^2 - p^2}\langle a_0(p)|\bar{s}s(0)|\rangle + \cdots$$
$$= \sin^2\varphi\frac{f_{\bar{n}na_0}^2 M_{a_0}^2}{M_{a_0}^2 - p^2} + \cos^2\varphi\frac{f_{\bar{s}sa_0}^2 M_{a_0}^2}{M_{a_0}^2 - p^2} + \cdots \quad (28)$$

Here we have used the following definitions:

$$\langle|\frac{\bar{u}u + \bar{d}d}{\sqrt{2}}(0) \mid f_0(p)\rangle = f_{\bar{n}nf_0}M_{f_0},$$
$$\langle|\frac{\bar{u}u - \bar{d}d}{\sqrt{2}}(0) \mid a_0(p)\rangle = f_{\bar{n}na_0}M_{a_0},$$
$$\langle|\bar{s}s(0) \mid f_0(p)\rangle = f_{\bar{s}sf_0}M_{f_0},$$
$$\langle|\bar{s}s(0) \mid a_0(p)\rangle = f_{\bar{s}sa_0}M_{a_0}. \quad (29)$$

After performing the standard manipulations of the quark–hadron duality (i.e. matching (27) to (28)) and Borel transformations, we can equate the coefficients of the $\sin^2\theta, \cos^2\theta, \sin^2\varphi$ and $\cos^2\varphi$, respectively. Finally we obtain the decay constants (coupling constants):

$$f_{\bar{n}nf_0} = f_{\bar{n}na_0} = 214 \pm 10\,\text{MeV},$$
$$f_{\bar{s}sf_0} = f_{\bar{s}sa_0} = 180 \pm 10\,\text{MeV},$$
$$f_{f_0} = \sqrt{\sin^2\theta f_{\bar{n}nf_0}^2 + \cos^2\theta f_{\bar{s}sf_0}^2},$$
$$f_{a_0} = \sqrt{\sin^2\varphi f_{\bar{n}na_0}^2 + \cos^2\varphi f_{\bar{s}sa_0}^2}. \quad (30)$$

The existing values for the mixing angle θ differ from each other greatly; the analysis of the J/ψ decays indicates $\theta = (34\pm6)^\circ$ or $\theta = (146\pm6)^\circ$ [26] while the analysis of the D_s^+ decays $D_s^+ \to f_0(980)\pi^+$ and $D_s^+ \to \phi\pi^+$ indicates $35^\circ \leq -\theta \leq 55^\circ$ [27]. If the value $\theta = (34\pm6)^\circ$ is taken, we can obtain $f_{f_0} = 191 \pm 13\,\text{MeV}$. The values for the decay constants $f_{\bar{n}nf_0}(f_{\bar{n}na_0})$ and $f_{\bar{s}sf_0}(f_{\bar{s}sa_0})$ are close to each other; the variations of θ and φ will not lead to significant changes for the net decay constants f_{f_0} and f_{a_0}, and in the following, we take the values $f_{f_0} = f_{a_0} = 191 \pm 13\,\text{MeV}$ for simplicity. This simplification will obviously introduce some imprecision; however, for the strong coupling constants $g_{f_0K^+K^-}(g_{a_0K^+K^-}) \sim \frac{1}{f_{f_0}}\left(\frac{1}{f_{a_0}}\right)$, and the final results will not be remarkably affected.

To obtain the above values in (30) for the two-point correlation functions in (25) and (26), the vacuum condensates are taken as $\langle\bar{s}s\rangle = 0.8\langle\bar{u}u\rangle$, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = (-240\pm10\,\text{MeV})^3$, $\langle\bar{s}\sigma\cdot Gs\rangle = (0.8\pm0.1)\langle\bar{s}s\rangle$, $\langle\bar{u}\sigma\cdot Gu\rangle = (0.8 \pm 0.1)\langle\bar{u}u\rangle$, $\langle\bar{d}\sigma\cdot Gd\rangle = (0.8 \pm 0.1)\langle\bar{d}d\rangle$, $M_{f_0} = M_{a_0} = 980\,\text{MeV}$. The threshold parameter s_0 is chosen to vary between 1.6–1.7 GeV^2 to avoid possible pollution from higher resonances and continuum states. In the region 1.2–2.0 GeV^2, the sum rules are almost independent of the Borel parameter M^2.

Now we return to the values of the strong coupling constants $g_{f_0K^+K^-}$, $g_{a_0K^+K^-}$ and choose the parameters as $m_s = 150\,\text{MeV}$, $f_{3K} = f_{3\pi} = 0.0035\,\text{GeV}^2$ at about $\mu = 1\,\text{GeV}$, $f_K = 0.160\,\text{GeV}$ and $M_K = 498\,\text{MeV}$ [21–23]. The duality thresholds in (23) and (24) are taken as $s_0 = 1.0$– $1.1\,\text{GeV}^2$ as determined from two-point K meson QCD sum rules to avoid possible pollution from the higher resonances and continuum states. The Borel parameters are chosen as $0.8 \leq M_1^2 \leq 1.6\,\text{GeV}^2$ and $2.0 \leq M_2^2 \leq 4.5\,\text{GeV}^2$, in those regions; the values for the strong coupling constants $g_{f_0K^+K^-}$ and $g_{f_0K^+K^-}$ are rather stable. Finally the numerical results for the strong coupling constants are obtained:

$$6.1 \leq g_{f_0K^+K^-}^{\bar{s}s}(g_{a_0K^+K^-}^{\bar{s}s}) \leq 7.5\,\text{GeV}; \quad (31)$$
$$4.4 \leq g_{f_0K^+K^-}^{\bar{n}n}(g_{a_0K^+K^-}^{\bar{n}n}) \leq 5.5\,\text{GeV}; \quad (32)$$
$$\theta = (34\pm6)^\circ \text{ [26]}, \quad g_{f_0K^+K^-} = 7.4 \sim 9.3;$$
$$\theta = (146\pm6)^\circ, \quad g_{f_0K^+K^-} = -4.0 \sim -1.8;$$
$$\theta = (-35 \sim -55)^\circ \text{ [27]}, \quad g_{f_0K^+K^-} = -0.2 \sim 3.0;$$
$$\theta = (-15 \sim -35)^\circ, \quad g_{f_0K^+K^-} = 3.0 \sim 5.8;$$
$$\varphi = (-30 \sim -40)^\circ, \quad g_{a_0K^+K^-} = 1.8 \sim 3.7$$
$$\varphi = 80^\circ, \quad g_{a_0K^+K^-} = 5.4 \sim 6.8;$$
$$\varphi = 90^\circ, \quad g_{a_0K^+K^-} = 4.4 \sim 5.5;$$
$$\varphi = 100^\circ, \quad g_{a_0K^+K^-} = 3.3 \sim 4.1. \quad (33)$$

From the above numerical results, in spite of the constituent structure differences between the $f_0(980)$ and $a_0(980)$ mesons, we can see that the strong couplings to the S-wave K^+K^- state through the $s\bar{s}$ components are larger than the corresponding ones through the $n\bar{n}$ components, $g_{f_0K^+K^-}^{\bar{s}s} \approx \sqrt{2}g_{f_0K^+K^-}^{\bar{n}n}$ and $g_{a_0K^+K^-}^{\bar{s}s} \approx \sqrt{2}g_{a_0K^+K^-}^{\bar{n}n}$. Due to the special Dirac structures of the interpolating currents J_{f_0} and J_{a_0}, the values of the strong K^+K^- couplings components of the $a_0(980)$ meson are about the same as the corresponding ones for the $f_0(980)$ meson, $g_{f_0K^+K^-}^{\bar{s}s} \approx g_{a_0K^+K^-}^{\bar{s}s}$, $g_{f_0K^+K^-}^{\bar{n}n} \approx g_{a_0K^+K^-}^{\bar{n}n}$. Furthermore, the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ are nearly linear functions of $\cos\theta$, $\sin\theta$, $\cos\varphi$ and $\sin\varphi$ (see (23) and (24)), and the variations with respect to the parameters θ and φ can change their values significantly, i.e., they are sensitive to the mixing angles.

In the following, we list the experimental data for the values of the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$. We have $g_{f_0K^+K^-} = 4.0\pm0.2\,\text{GeV}$ by the KLOE Collaboration [28], $g_{f_0K^+K^-} = 4.3\pm0.5\,\text{GeV}$ by the CMD-2 Collaboration [29], $g_{f_0K^+K^-} = 5.6 \pm 0.8$ by the SND Collaboration [30], $g_{f_0K^+K^-} = 2.2 \pm 0.2$ by the WA102 Collaboration [31], $g_{f_0K^+K^-} = 0.5 \pm 0.6$ by the E791 Collaboration [32], $g_{a_0K^+K^-} = 2.3 \pm 0.7\,\text{GeV}$ by the KLOE Collaboration [33] and $g_{a_0K^+K^-} = 2.63^{+1.84}_{-1.28}\,\text{GeV}$ by the analysis of the KLOE Collaboration data [34]. While the theoretical values are $g_{f_0K^+K^-} = 2.24\,\text{GeV}$ by the linear sigma model [35], we have $g_{f_0K^+K^-} = 3.68\pm0.13\,\text{GeV}$ and $g_{a_0K^+K^-} = 5.50\pm0.11\,\text{GeV}$ by unitary chiral perturbation theory [36].

Comparing with all the controversial values, we cannot reach a general consensus on the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$. If we take the mixing angle $\theta = -15^\circ \sim -35^\circ$ for the $f_0(980)$ meson, the value of the strong coupling constant $g_{f_0K^+K^-}$ is $g_{f_0K^+K^-} = 3.0 \sim 5.8$, which is considerably more compatible with the existing experimental data. For the $a_0(980)$ meson, no conclusion can be drawn from the existing values for the mixing angle φ. A precise determination of those values calls for more accurate measures and original theoretical approaches. Whatever the mixing angles θ, φ may be, we observe that the strong couplings through both the $n\bar{n}$ and $s\bar{s}$ components are remarkably large. This fact obviously supports the hadronic dressing mechanism; the $f_0(980)$ and $a_0(980)$ mesons can be taken to have small $q\bar{q}$ kernels of typical meson size with a large virtual S-wave $K\bar{K}$ cloud.

3 Conclusions

In this article, with the assumption of explicit isospin violation arising from $f_0(980)$–$a_0(980)$ mixing, we take the point of view that the $f_0(980)$ and $a_0(980)$ mesons have both strange and non-strange $q\bar{q}$ components, and we evaluate the strong coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ within the framework of the light-cone QCD sum rules approach. Taking into account the controversial values that emerge from different experimental and theoretical determinations, we cannot reach a general consensus. Our observation concerning the large scalar–KK coupling constants $g^{\bar{n}n}_{f_0K^+K^-}$, $g^{\bar{s}s}_{f_0K^+K^-}$, $g^{\bar{n}n}_{a_0K^+K^-}$ and $g^{\bar{s}s}_{a_0K^+K^-}$ based on the light-cone QCD sum rules approach will support the hadronic dressing mechanism; furthermore, in spite of the constituent structure differences between the $f_0(980)$ and $a_0(980)$ mesons, the strange components have larger strong coupling constants with the K^+K^- state than the corresponding non-strange ones, $g^{\bar{s}s}_{f_0K^+K^-} \approx \sqrt{2}g^{\bar{n}n}_{f_0K^+K^-}$ and $g^{\bar{s}s}_{a_0K^+K^-} \approx \sqrt{2}g^{\bar{n}n}_{a_0K^+K^-}$.

Note added. The interest in the nature of the light scalar $f_0(980)$ and $a_0(980)$ mesons and their mixing was renewed recently. There have been a number of articles attempting to elucidate those elusive mesons since we have finished our article; for example, [37].

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